Complex Analysis Prelim Exam
UC Department of Math
Spring 26

Only answer 5 out of the 6 questions. Choose and indicate clearly which 5 you have chosen.

1. Let $f(z) = x^2 + y^2 + 2ixy$, where $z = x + iy$. Find the values of z where $f'(z)$ exists and calculate $f'(z)$ there.

2. Determine the number of zeroes of the function $f(z) = e^{2z-4} + z^3 + 2z - 1$ in the disc $\Omega = \{|z| < 3\}$.

3. Evaluate the integral
$$\int_0^\infty \frac{\sin(4x)}{x(x^2+1)}\,dx.$$

4. Let $\Omega = \{z \in \mathbb{C} \colon |z| < 1\} \setminus [0, 1)$ be the unit disk with a radial slit along the positive real axis. Construct a conformal map from Ω to the upper half-plane. (Hint, consider a branch of square root of z as part of your solution)

5. Let Ω be a planar domain and suppose that $f : \mathbb{D} \to \Omega$ is a conformal map. Prove that $f'(z) \neq 0$ for each $z \in \mathbb{D}$.

6. Let f be a complex analytic function on a domain $\Omega \subset \mathbb{C}$, and $z_0 \in \Omega$. Suppose that $r > 0$ for which the disk centered at z_0 with radius $2r$ is contained in Ω. Let $\gamma : [0, 2\pi] \to \Omega$ be given by $\gamma(t) = z_0 + r\,e^{it}$. Compute
$$\frac{1}{2\pi i} \int_\gamma \frac{\zeta f(\zeta)}{(\zeta - z_0)^2}\,d\zeta$$
in purely in terms of z_0, $f(z_0)$, and the derivatives of f at z_0.