



Calculus: *Limit Laws*

Special Limits

$$\lim_{x \rightarrow a} c = c$$

$$\lim_{x \rightarrow a} x = a$$

Common Limit Laws

1 Sum/Difference Law

$$\lim_{x \rightarrow a} f(x) \pm g(x) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} 3x^2 + 2x = \lim_{x \rightarrow a} 3x^2 + \lim_{x \rightarrow a} 2x$$

2 Constant Law: c is a constant

$$\lim_{x \rightarrow a} c * f(x) = c * \lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow a} 3x^2 = 3 * \lim_{x \rightarrow a} x^2$$

3 Product Law

$$\lim_{x \rightarrow a} f(x) * g(x) = \lim_{x \rightarrow a} f(x) * \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} x \sin(x) = \lim_{x \rightarrow a} x * \lim_{x \rightarrow a} \sin(x)$$

4 Quotient Law

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} (x)}{\lim_{x \rightarrow a} (x)}$$

$$\lim_{x \rightarrow a} \frac{x^2 + 1}{x - 1} = \frac{\lim_{x \rightarrow a} x^2 + 1}{\lim_{x \rightarrow a} x - 1}$$

5 Exponent Law: n is a positive integer

$$\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} (x)]^n$$

$$\lim_{x \rightarrow a} x^2 = [\lim_{x \rightarrow a} x]^2$$

Direct Substitution Property

If $f(x)$ is a polynomial or rational function and a is in the domain of $f(x)$ then,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Example: $\lim_{x \rightarrow 2} 3x^2 + 2x$

$$\begin{aligned} & \stackrel{(1)}{=} \lim_{x \rightarrow 2} 3x^2 + \lim_{x \rightarrow 2} 2x \stackrel{(2)}{=} 3 \lim_{x \rightarrow 2} x^2 + 2 \lim_{x \rightarrow 2} x \stackrel{(5)}{=} 3[\lim_{x \rightarrow 2} x]^2 + 2 \lim_{x \rightarrow 2} x \stackrel{\text{DS}}{=} 3[2]^2 + 2[2] = 16 \end{aligned}$$