m^2 = *math minute*

Calculus: *Concavity & The Second Derivative*

Concavity

- A function f is **concave up** on an interval (a, b) if f' is an *increasing* function on the interval (a, b).
- A function f is **concave down** on an interval (a, b) if f' is a *decreasing* function on the interval (a, b).

We are able to find if a function f is *increasing* or *decreasing* by using the derivative. Therefore, to find if f' is *increasing* or *decreasing*, we must use the derivative of f' also known as the **second derivative** f''.

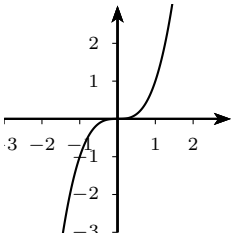
Concavity Test Using The Second Derivative

- If $f''(x) > 0$ on the interval (a, b) then $f(x)$ is **concave up** on the interval (a, b).
- If $f''(x) < 0$ on the interval (a, b) then $f(x)$ is **concave down** on the interval (a, b).

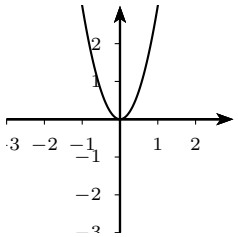
Inflection Point

- A point which the graph of a function f changes concavity is called an **infelction point** of the function f.
  - If an **inflection point** of f occurs at a point $(a, f(a))$ then $f''(a) = 0$.
  - Note: The converse is **not** true. That is if $f''(a) = 0$, this does **not** imply that an *inflection point* of the function f occurs at a point $(a, f(a))$.

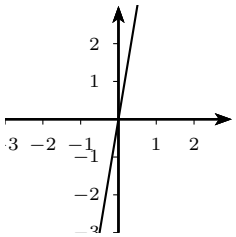
Example



$f(x) = x^3$



$f'(x) = 3x^2$



$f''(x) = 6x$

- As $f''(x) = 6x$ and $6x < 0$ on the interval $(-\infty, 0)$ then the original function $f(x) = x^3$ is **concave down** on the interval $(-\infty, 0)$.
- As $f''(x) = 6x$ and $6x > 0$ on the interval $(0, \infty)$ then the original function $f(x) = x^3$ is **concave up** on the interval $(0, \infty)$.
- As the function f switches concavity at $x = 0$ (*down to up*), then an **inflection point** of the function f occurs at $(0, 0)$.