Can a Flow-Network Approach Shed Light on Children's Problem Solving?

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Can a Flow-Network Approach Shed Light on Children’s Problem Solving?

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A complete theory of human behavior must capture both the apparent randomness of behavior (its strong context dependence) as well as its stability (a surprising resistance to changes in behavior despite salient changes in the context). Recent efforts in cognitive science have made important discoveries toward such a theory, emphasizing the idea that skilled behavior seeks to balance overregular tendencies with tendencies that are overrandom. The hallmark of these efforts is the idea of self-organized criticality, the state of a system poised toward maximally adaptive behavior, characterized by pink-noise pattern of variability. In this article, we expand these efforts, looking for a new measure to capture the balance between order and disorder, one that can be applied to small data sets of categorical performance. The proposed measure borrows ideas from information theory, previously applied to the stability of energy flow in an ecosystem. Using published data on a problem-solving task with preschoolers, we describe ways in which this measure could be applied. Results are promising, opening the possibility for studying the trade-off between randomness and stability in children’s reasoning.

Picture a child deciding which of two objects would sink faster in water. The child might not know what feature to pay attention to. Indeed, with sufficient

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variability in object features, responses are likely to follow a random pattern (e.g., Inhelder & Piaget, 1958). There are circumstances, however, in which children perform consistently: They pick the heavier object repeatedly, ignoring other features of the objects (e.g., Kloos & Somerville, 2001; C. Smith, Carey, & Wiser, 1985). In fact, children sometimes perform consistently even after negative feedback: despite making mistakes, they fail to change their behavior. And there is one more complication: sometimes performance is marked by strong context dependence. A child might focus on the mass of an object under some circumstances but then focus on density after only a slight change in context (Kloos, Fisher, & Van Orden, 2010; Kohn, 1993).

The set of behaviors displayed in this problem-solving task illustrates the complexity of beliefs: there is an apparent stability of thought on the one hand, a stability that persists even through highly relevant changes in the context. And then there is an apparent randomness of thought, where it seems as though everything matters. The stability pattern of behavior has prompted numerous attempts to describe the specifics of the mental content that supposedly explains stable behavior: the content of representations, schemas, beliefs, strategies, heuristics, and goals. However, given the strong context dependence of stability, these efforts might be inadequate. A more promising theory of cognition would anticipate both the temporary randomness of behavior and its temporary stability.

Advances in cognitive science have made the necessary jump to embrace the two sides of the coin: the strong context dependence on the one hand and the apparent regularity of behavior on the other hand (cf. L. B. Smith, 2005; L. B. Smith & Jones, 1993). They acknowledge that adaptive behavior requires the interplay between order and disorder, a balance that is reached when a system settles in a state of self-organized criticality (e.g., Kloos & Van Orden, 2010; Van Orden, Kloos, & Wallot, 2011). Systems that display self-organized criticality (physical, biological, or social) are neither fully stable nor fully random. They comprise of a multitude of interacting components and as such are attracted to a critical state, a state in which forces of stability balance those of randomness (cf., e.g., Bak, 1996; Jensen, 1998; Kelso, 1995).

The degree to which a system approaches self-organized criticality can be captured by the fractal structure of variability (e.g., Holden, 2005; Holden, Van Orden, & Turvey, 2009; Kloos & Van Orden, 2010; Riley & Turvey, 2002; Van Orden & Holden, 2002; Van Orden, Holden, & Turvey, 2003, 2005; Van Orden et al., 2011). However, the proposed mapping between fractal structure and degree of self-organized criticality is not without difficulty, at least on methodological grounds. This is because fractal analyses require large data sets consisting of several hundred trials and measuring behavior on a continuous scale. Such data are not always available, especially not from reasoning tasks with children. For example, the reasoning task with sinking object described earlier cannot feasibly be carried out over hundreds of trials, and the answer
options have little variability. How could such a task lend itself to capturing relative stability of behavior? Motivated by extensive discussions with Guy Van Orden, we describe such a measure here using the flow-network formalism developed by Ulanowicz (1986, 1998, 2000, 2009a, 2009b, 2011). In what follows, this formalism is described briefly.

ORDER AND DISORDER IN THE FLOW NETWORK OF AN ECOSYSTEM

Ecosystems can be characterized by the cycle of energy that is transferred among species of a circumscribed niche. These systems can emerge, develop, and transform over time as a function of factors such as the kinds of interacting species, the availability of energy, and the dynamics of energy dissipation (Ulanowicz, 1986). Consider, for example, the ecosystem of cone springs, a model system that consists of plants, detritus, bacteria, detritivores, and carnivores. Figure 1 shows a schematic of these components with a simplified version of the energy that they exchange (in kilo calories): Plants supply energy to the system by decomposing into organic material (detritus). In turn, detritus provides the nutrition for bacteria and detritivores (e.g., worms). Bacteria also feed detritivores, which in turn feed carnivores. And both detritivores and carnivores turn into detritus, thus starting the cycle all over again.

FIGURE 1  Schematic of the energy flow in a cone-spring system. The system is supplied by an energy source (Component 1), which recycles through another four components of the system (Components 2–5; adapted from Ulanowicz, 2000).
The flow-network approach to ecosystems makes it possible to combine order and disorder in a system’s structure in a quantitative way. Disorder in an ecosystem is conceptualized as the overhead of energy, energy lost due to the redundancy of links. And order is conceptualized as the efficiency by which energy is recycled, sustained through constrained energy flow. Figure 2 shows three schematics of systems that differ in order versus disorder. In the maximally redundant case (Figure 2a), energy flow is minimally constrained, yielding high energy loss and high resistance to perturbation. In contrast, in the maximally efficient case (Figure 2c), energy flow is ordered nonredundantly so that the energy loss is minimal. In this latter case, the system is highly susceptible to perturbations. The ideal system strives toward a balance between order and disorder (schematically shown in Figure 2b). In this case, energy links among species are not too constrained or too redundant, guaranteeing maximal robustness and adaptability (Ulanowicz, 2000).

The fluctuation between disorder and order in a network can be captured by information-theory concepts, including uncertainty, joint probability, conditional probability, and degree of order (Ulanowicz, 1986, 2009a, 2009b, 2011). The general idea is that the degree of articulation in a flow network (i.e., its degree of efficiency) is inversely related to uncertainty. Without an articulating structure, the network flow is random, making it difficult to predict its trajectory. In contrast, in a better articulated flow network, the spectrum of possible flow trajectories is reduced, making it possible to predict a future state. Degree of uncertainty can be estimated from observations of past outcomes of similar (or associated) events. In the absence of any regularity, the uncertainty is maximal. When there is some degree of interconnection among components of a system, uncertainty decreases on the basis of increased conditional probabilities. The
resulting index, referred to average mutual information, can then be used to estimate the degree of order, a measure of how much of the uncertainty is accounted for. Could the same idea be applied to data from children’s problem solving?

FLOW NETWORKS IN CHILDREN’S PROBLEM SOLVING

We propose that problem-solving performance can be captured in a flow network, the components of the network consisting of performance in a trial or performance on a subset of trials. To construct the flow network, consider performance in the problem-solving task with sinking objects described earlier. Trials consist of having to decide which of two objects would sink fastest in water. And trials could differ in how individual object features (e.g., mass, volume, density) covary. One can envision a kind of pair in which one object is heavier, larger, and denser than the other (i.e., mass and density, as well as volume and density, covary positively in this pair). In another kind of pair, one object could be heavier, smaller, and denser than the other (i.e., mass and density covary positively, whereas volume and density covary negatively). And in another kind of pair, one object could be lighter, smaller, and denser than the other (i.e., mass and density, as well as volume and density, covary negatively in this pair of objects). Figure 3 shows example pairs for each of these kinds of trials. Average performance on a kind of trial could serve as a component (or node) of the flow network.

If children hold a stable belief, performance on one trial should predict performance on another trial (and performance on a subset of trials should predict performance on another subset of trials). Take, for example, the belief that the

![Figure 3](https://example.com/figure3.png)

**FIGURE 3** Example pairs of sinking objects of different sizes and different number of weights. In each pair, the faster sinking object is underlined. In Subset A, the bigger and heavier object sinks fastest; in Subset B, the smaller and heavier object sinks faster; in Subset C, the smaller and lighter object sinks faster. (color figure available online)
heavier object sinks fastest, a common (although incorrect) belief (e.g., C. Smith et al., 1985). In this case, children should perform consistently well on trials in which heavy objects sink fastest, whereas they should perform consistently low on trials in which heavy objects sink slowly (namely, when heavy objects are very large). Such stability (i.e., consistency) in performance is expected to have low uncertainty and high degree of order. In other words, the degree of order should be highest when performance is stable (i.e., consistent across trials) and lowest when performance follows a random pattern.

In order to test these predictions, we reanalyzed a data set obtained from preschoolers participating in a sinking-objects prediction task (Baker, Haussmann, Kloos, & Fisher, 2011). In this task, children were presented with 24 pairs of objects, 1 pair at a time, and asked to decide which of the two objects would sink faster. The trials differed in how mass, volume, and density covaried within a pair of objects, yielding three types of trials, eight trials per type (cf. Figure 3). The crucial manipulation took place in a preliminary phase in which children were either given information about density (experimental condition) or not (control condition). It is important to note that overall performance accuracy did not differ between conditions. The difference between conditions was only in terms of consistency (vs. random responding). Children in the experimental condition came to believe that the heavier of two objects would always sink faster (performing above chance on critical trials and below chance on the other trials). In contrast, responses in the control condition were random (performing correctly on some trials and incorrectly on others). Given the difference in performance consistency between experimental and control condition, this data set provides an ideal contrast to apply the measures of uncertainty, average mutual information, and degree of order. The prediction is that performance in the experimental condition (relative to performance in the control condition) yields low uncertainty and high degree of order.

In what follows, we explain our calculations of average uncertainty \( (H) \), average mutual information \( (AMI) \), and degree of order \( (AMI/H) \) for the Baker et al. (2011) data set. We first focus on the flow network established by the subsets of trials (eight trials per subset; three subsets total). Each trial has two answer options (pick correct object vs. pick incorrect object), yielding nine possible outcomes across each subset of trials (0, 1, 2, 3, etc., up to 8 trials correct). Average uncertainty \( H \) was calculated for each subset following the equation

\[
H = -K \sum p_i \log_2(p_i),
\]

where \( p_i = 1/n \), and \( n \) represents the total number of answer options.\(^1\)

\(^1\)The base of the logarithm was chosen to be 2 (i.e., 2 bits of information), and the value of \( K \) was chosen to be the unity value of 1, following the suggestion of Ulanowicz (1986, 2000).
Figure 4 shows the obtained values of $H$ for each subset of trials, separated by condition. As predicted, $H$ was lower in each subset of the experimental group compared with the subsets of the control group. In fact, one of the subsets in the control group (Subset C) yielded a value of $H$ that was close to maximal uncertainty $H_{\text{max}} = 3.17^2$, the level of uncertainty obtained when there is no a priori information about the distribution of answer options.

The next step was to calculate $\text{AMI}$ for each directional dyad of subsets (e.g., Subset A and Subset B) using joint probabilities of all possible dyads of subsets $[p(a_j, b_i)]$ and determining the conditional probabilities of the directional links between the two subsets $[p(b_i | a_j)]$. The pertinent formula was

$$\text{AMI} = K \sum_i \sum_j p(a_j, b_i) \log_2[p(b_i | a_j) / p(b_i)].$$

Obtained $\text{AMI}$ values (one for each direction of the subset dyad) are shown in the parentheses of the directional links in Figure 4. Note that $\text{AMI}$ values differ as a function of direction in the experimental condition but not in the control condition. For example, whereas $\text{AMI}$ of Subset A given Subset B is larger than $\text{AMI}$ of Subset B given Subset A in the experimental condition (1.05 vs. 0.88), the corresponding $\text{AMI}$ values are identical in the control condition (1.20). This difference of $\text{AMI}$ as a function of direction reflects children’s sensitivity to the temporal sequence of trials when performance is nonrandom. In the case in which performance follows random patterns (control condition), this sensitivity to timing is missing.

$^2H_{\text{max}} = K \log_2 n.$
The last step was to determine the proportion of degree of order for each directional link between subset dyads, reflected as ratio between average mutual information and uncertainty (Ulanowicz, 2009a). Figure 4 shows the proportions for each of the directional links between subsets. For the experimental condition, results show that degree of order was higher between subsets for which mass and density covaried positively (Subsets A and B) than for dyads that included Subset C (for which mass and density covaried negatively). This is because uncertainty for Subset C differed substantially from that of Subsets A and B. In the control condition, on the other hand, uncertainty and degree of order was comparable across subsets and dyads of subsets. Surprisingly, unlike what we predicted, degree of order was not higher in the experimental than the control condition.

To follow up on these findings, we conducted the same analyses of $H$, $AMI$, and $AMI/H$ for a flow network that consists of individual trials as components. Recall that a trial only had two answer options, thus yielding $H_{\text{max}} = 1$. Figure 5 shows the results of $H$ for each trial (out of 24), separated by condition. Analogous to our findings across subsets, $H$ was higher in the control condition (solid line in Figure 5) than the experimental condition (dashed line). In fact, $H$ was very close to the maximal uncertainty in the control condition, fluctuating little across trials. $H$ for trials in the experimental condition fluctuated rather substantially with lower values in trials of Subsets A and B and higher values in trials of Subset C.

FIGURE 5 Average uncertainty ($H$) per trial in each subset, separated for experimental and control condition.
FIGURE 6 Proportion of degree of order \((AMI/H)\) for pairs of trials, shown as color-coded quartiles. White boxes represent the lowest quartile \((< .0316)\), the darkest boxes represent the highest quartile \((> .2930)\), and the two gray scales represent the two middle quartiles \((.0316 \text{ to } .1268 \text{ and } .1269 \text{ to } .2930)\). The diagonal is blacked out. The triangle above the diagonal shows the pairwise proportions obtained for trials in the experimental condition, and the triangle below the diagonal shows the pairwise proportions obtained for trials in the control condition.

Following the same calculations of \(AMI/H\) described earlier, Figure 6 shows the obtained results for each pair of trials (separated by condition).\(^3\) The top half of the \(24 \times 24\) matrix shows the degree of order for each dyad of trials obtained in the experimental condition. And the bottom half of the matrix shows the degree of order for each dyad of trials obtained in the control condition. The diagonal of the matrix is blacked out. The degree of order \((AMI/H)\) is color coded to represent each quartiles of the degree-of-order distribution. White stands for the lowest quartile, dark gray stands for the highest quartile, and the two intermediate grays stand for the two intermediate quartiles. For illustration purposes, consider

\(^3\)The figure shows only one of the two directions for each dyad of trials (e.g., Trial 17 given Trial 1), not both. This is because there was no difference in direction.
Area (c) of the experimental condition (top right square). It illustrates the degree of order for each trial in Subset C with each trial in Subset A (in one direction only). The degree of order in the 1–17 dyad was in the lowest quartile (white), whereas the degree of order in the 3–19 dyad was in the highest quartile (dark gray).

Comparing the top and bottom triangular areas, findings show, first, that the proportion of degree of order was higher in the experimental than the control condition, \( \chi^2 = 9.91, df = 3, p = .019 \). With some exceptions, the same trend was observed when single areas were compared (whether areas represented dyads of trials within the same subset [Area (f)] or whether areas represented dyads of trials that cross subsets [Areas (b), (c), (e)]), \( \chi^2 > 8.77, df = 3, p < .03 \). The exception was for areas (a) and (d), where there was no difference between experimental and control condition, \( p > .25 \). Taken together, findings on the basis of trials were in line with our predictions of how the two conditions differ: Trial-by-trial performance in the experimental condition appeared more interconnected, making it possible to estimate information about a specific trial on the basis of information from other trials. In contrast, the control group had a high level of uncertainty, and trial-by-trial performance was only loosely interrelated, with lower degree of order.

**CONCLUSION AND FURTHER DIRECTION**

Our goal was to apply a flow-network approach to problem-solving data in order to evaluate whether information theory could provide a measure of relative stability of behavior. This approach was previously applied to ecological niches, systems in which order (vs. disorder) is seen as articulation (vs. redundancy) of energy flow. Problem-solving behavior might not have the same flow of energy between components. Nevertheless, relative order is likely to be similarly adaptive for all complex systems, opening the door for these ideas to be applied to cognitive systems. Preliminary findings are in support of our hypothesis: Measures of uncertainty were lower for system components that were more stable, and vice versa, measures of degree of order were higher in these cases. In other words, our calculations were sensitive to the degree of structure in the organization of events of the experimental condition, at least in the case in which components pertained to individual trials.

Note that accuracy level is not sufficient to determine the articulation and relative stability of problem-solving behavior. This is because stability in performance can stem both from correct beliefs (e.g., the belief that density determines an object’s sinking rate) and from mistaken beliefs (e.g., the belief that mass determines sinking rate). A group of participants can obtain low accuracy but still have high levels of articulation among responses.
The tools described here specify a link between actor and environment that is precise enough to address an apparent dualism: that organization of the system seems extremely stable in some cases, unaffected by even relevant changes in the context, and extremely fluid in other cases, affected by even miniscule changes in the context. Ideas from complexity science, applied to the energy flow in ecosystems, could shed light on this apparent dualism. Applied to children’s problem solving, complexity science could provide an explanation of why a child’s belief resists change in some circumstances and not in others. In fact, it could predictively determine circumstances in which a child’s belief loses strength and is replaced with a new one. In other words, the theory of complex systems can conceptualize the dualism of stability and fluidity in a child’s thought as well as propose a parameter that controls belief stability.

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